**Theorem.** *Let $n \in \mathbb{Z}^+$. Let $\Phi_n(x) := \prod_{a \in (\mathbb{Z}/n)^\times}(x - e_n(a)) \in \mathbb{Z}[x]$.*[1] *Then: $\Phi_n$ is irreducible.*

*Proof.* Let $f \in \mathbb{Z}[x]$ be such that $f(e_n(1)) = 0$. Let $a \in (\mathbb{Z}/n)^\times$. By Dirichlet's theorem there are infinitely many primes $p \equiv a \pmod{n}$. For each such $p$ let $\mathfrak{p}$ be a prime of $\mathbb{Q}(\zeta_n)$ above $p$. Because $f(x^p) \equiv f(x)^p \pmod{\mathfrak{p}}$ it follows that $\mathfrak{p} \mid f(\zeta_n^a)$. Thus $f(\zeta_n^a) = 0$ and so $\Phi_n \mid f$.

[1] Here $e_n(a) := e^{\frac{2\pi i a}{n}}$ as usual.